Chapter 7: Section 7.1

In problems 1-6 use definition 7.1.1 to find $\mathcal{L}\{f(t)\}$:

1. $f(t) = e^t \sin(3t)$
2. $f(t) = \begin{cases} e^{2t} & \text{if} \quad 0 < t < 3 \\ 1 & \text{if} \quad 3 < t \end{cases}$
3. $f(t) = \begin{cases} 1 - t & \text{if} \quad 0 < t < 1 \\ 0 & \text{if} \quad 1 < t \end{cases}$
4. $t \cos t$
5. e^{-3t+2}
6. $t^2 e^{-t}$

In problems 7-12 use theorem 7.1.1 to find $\mathcal{L}\{f(t)\}$:

7. $f(t) = 3\cosh(2t) - t^2 + 8$
8. $f(t) = \sin(7t)\cos(3t)$
9. $f(t) = (t+3)^2$
10. $f(t) = (e^{3t} + e^{-3t})^2$
11. $f(t) = e^t \cosh(3t)$
12. $f(t) = \cos(3t + \pi/3)$

Chapter 7: Section 7.2

In problems 1-9 find the given inverse Laplace transform $\mathcal{L}^{-1}\{f(t)\}$:

1. $\mathcal{L}^{-1}\left\{\dfrac{(s-1)^2}{s^4}\right\}$
2. $\mathcal{L}^{-1}\left\{\dfrac{1}{6s+3}\right\}$
3. $\mathcal{L}^{-1}\left\{\dfrac{1}{9s^2+1}\right\}$
4. $\mathcal{L}^{-1}\left\{\dfrac{9s}{s^2+4}-\dfrac{1}{s}+\dfrac{1}{s+3}\right\}$
5. $\mathcal{L}^{-1}\left\{\dfrac{2s+1}{s^2+2}\right\}$
6. $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+2)(s^2+4)}\right\}$
7. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-s+20}\right\}$
8. $\mathcal{L}^{-1}\left\{\dfrac{3s^2+5s+3}{s^3+s^2}\right\}$
9. $\mathcal{L}^{-1}\left\{\dfrac{s^2-26s-47}{(s-1)(s+2)(s+5)}\right\}$

In problems 10-14 use Laplace transform to solve the given initial value problem:

10. $3y' - y = 0,\ y(0) = 9$
11. $y' - y = e^{3t},\ y(0) = 2$
12. $y'' + y = t^2 + 2,\ y(0) = 1,\ y'(0) = -1$
13. $y'' - 7y' + 10y = 9\cos t + 7\sin t,\ y(0) = 5,\ y'(0) = -4$
14. $y''' - y'' + y' - y = 0,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -1$

Chapter 7: Section 7.3

Solve the following problems:

1. $\mathcal{L}\{\sin(3t)(5+e^{2t}-e^{-3t})\}$
2. $\mathcal{L}\{e^{2t}(-7+t^2-2\cos(t/2))\}$
3. $\mathcal{L}^{-1}\left\{\dfrac{3}{2s^2+8s+10}\right\}$
4. $\mathcal{L}^{-1}\left\{\dfrac{3s+2}{s^2+2s+10}\right\}$
5. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+6s+9}\right\}$
6. $\mathcal{L}^{-1}\left\{\dfrac{7}{(s+3)^3}\right\}$
7. $\mathcal{L}^{-1}\left\{\dfrac{4s-2}{s^2(s+1)^3}\right\}$
8. $\mathcal{L}^{-1}\left\{\dfrac{2s}{(s-3)^3}\right\}$
9. $\mathcal{L}^{-1}\left\{\dfrac{-2s^2+8s-14}{(s+1)(s^2-2s+5)}\right\}$

In problems 10-13 use Laplace transform to solve the given initial value problem:

10. $y'-y=1+te^t$, $y(0)=0$
11. $y''-2y'+5y=-8e^{-t}$, $y(0)=2$, $y'(0)=12$
12. $y''+6y'+9y=0$, $y(0)=-1$, $y'(0)=6$
13. $2y''+20y'+51y=0$, $y(0)=2$, $y'(0)=0$

Chapter 7: Section 7.4

In problems 1-4 use Theorem 7.4.1 to evaluate the given Laplace transform:

1. $\mathcal{L}\{te^{-5t}\}$
2. $\mathcal{L}\left\{t\cos(\sqrt{2}t)\right\}$
3. $\mathcal{L}\{t^2\sinh 3t\}$
4. $\mathcal{L}\{te^{-t}\sin(4t)\}$

Use Laplace transform to solve the given initial value problem:

5. $y' + y = 2t\sin t,\ y(0) = 0$

In problems 6-9 use Theorem 7.4.2 to evaluate the given Laplace transform:

6. $\mathcal{L}\left\{\int_0^t \cos\tau d\tau\right\}$
7. $\mathcal{L}\left\{\int_0^t \tau\sin\tau d\tau\right\}$
8. $\mathcal{L}\left\{\int_0^t \tau e^{t-\tau} d\tau\right\}$
9. $\mathcal{L}\left\{t\int_0^t \tau e^{-\tau} d\tau\right\}$

In problems 10-12 use formula (8) to evaluate the given inverse transform:

10. $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s-1)}\right\}$
11. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3(s-1)}\right\}$
12. $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s-3)^2}\right\}$